

# The algorithms bundle\*

Rogério Brito  
rbrito@ime.usp.br

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## Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>	3.11 Printing Messages . . . . .	8
<b>2</b>	<b>Installation</b>	<b>2</b>	3.12 Comments . . . . .	9
<b>3</b>	<b>Environment:</b> <code>algorithmic</code>	<b>2</b>	3.13 An Example . . . . .	9
3.1	The Simple Statement . . . . .	3	3.14 Options/Customization	10
3.2	The <i>if-then-else</i> Statement . . . . .	3	<b>4</b> <b>Environment:</b> <code>algorithm</code>	<b>14</b>
3.3	The <i>for</i> Loop . . . . .	4	4.1 General . . . . .	14
3.4	The <i>while</i> Loop . . . . .	5	4.2 An Example . . . . .	14
3.5	The <i>repeat-until</i> Loop . . . . .	5	4.3 Options . . . . .	15
3.6	The Infinite Loop . . . . .	6	4.4 Customization . . . . .	15
3.7	The Logical Connectives . . . . .	6	<b>5</b> <b>References in Algorithms</b>	<b>16</b>
3.8	The Precondition . . . . .	7	<b>6</b> <b>Known Issues</b>	<b>17</b>
3.9	The Postcondition . . . . .	7	<b>7</b> <b>General Hints</b>	<b>18</b>
3.10	Returning Values . . . . .	8		

## List of Algorithms

1	Calculate $y = x^n$ . . . . .	15
2	Calculate $y = x^n$ . . . . .	17

## 1 Introduction

This package provides two environments, `algorithmic` and `algorithm`, which are designed to be used together but may, depending on the necessities of the user, be used separately.

---

\*This document corresponds to `algorithms` v0.1, dated 2009/08/24.

The `algorithmic` environment provides an environment for describing algorithms and the `algorithm` environment provides a “float” wrapper for algorithms (implemented using `algorithmic` or some other method at the user’s option). The reason for two environments being provided is to allow the user maximum flexibility.

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Currently, this package consists of the following files:

- `algorithms.ins`: the driver file
- `algorithms.dtx`: the source file
- `COPYING`: the license file
- `README`: remarks about the package
- `THANKS`: mentions of thanks for contributors to the package

Starting with the 2009-08-24 release, the package is now versioned and this document corresponds to version v0.1.

If you use this package, the author would kindly appreciate if you mentioned it in your documents, so as to let the package be better known and have more contributors, to make it better for the community itself. This is *not* required by the license: it’s just a friendly request.

## 2 Installation

The installation procedure of `algorithms` follows the usual practice of packages shipped with a pair of `.ins`/`.dtx`—simply type the command:

```
latex algorithms.ins
```

and the `.sty` files will be generated. Copy them to a place that is referenced by your `LATEX` distribution. To generate the documentation, type:

```
latex algorithms.dtx
```

## 3 The `algorithmic` Environment

Within an `algorithmic` a number of commands for typesetting popular algorithmic constructs are available. In general, the commands provided can be arbitrarily nested to describe quite complex algorithms. An optional argument to the `\begin{algorithmic}` statement can be used to turn on line numbering by giving a positive integer indicating the required frequency of line numbering. For example, `\begin{algorithmic}[5]` would cause every fifth line to be numbered.

### 3.1 The Simple Statement

The simple statement takes the form

```
\STATE <text>
```

and is used for simple statements. For example,

```
\begin{algorithmic}
    \STATE \$S \leftarrow 0$
\end{algorithmic}
```

would produce

```
S ← 0
```

With line numbering selected for every line, using,

```
\begin{algorithmic}[1]
    \STATE \$S \leftarrow 0$
\end{algorithmic}
```

we would get

```
1: S ← 0
```

**Warning** *For users of earlier versions of `algorithmic` this construct is a cause of an incompatibility. In the earlier version, instead of starting simple statements with the `\STATE` command, simple statements were entered as free text and terminated with `\|` command. Unfortunately, this simpler method failed to survive the modifications necessary for statement numbering. However, the `\|` command can still be used to force a line break within a simple statement.*

### 3.2 The *if-then-else* Statement

The *if-then-else* construct takes the forms:

```
\IF{<condition>} <text> \ENDIF
\IF{<condition>} <text1> \ELSE <text2> \ENDIF
\IF{<condition1>} <text1> \ELSIF{<condition2>} <text2> \ELSE <text3> \ENDIF
```

In the third of these forms there is no limit placed on the number of `\ELSIF{<condition>}` that may be used. For example,

```
\begin{algorithmic}
\IF{some condition is true}
\STATE do some processing
\ELSIF{some other condition is true}
\STATE do some different processing
\ELSIF{some even more bizarre condition is met}
```

```
\STATE do something else
\ELSE
\STATE do the default actions
\ENDIF
\end{algorithmic}
```

would produce

```
if some condition is true then
    do some processing
else if some other condition is true then
    do some different processing
else if some even more bizarre condition is met then
    do something else
else
    do the default actions
end if
```

with appropriate indentations.

### 3.3 The *for* Loop

The *for* loop takes two forms. Namely:

```
\FOR{<condition>} <text> \ENDFOR
\FORALL{<condition>} <text> \ENDFOR
```

For example,

```
\begin{algorithmic}
\FOR{$i=0$ to $10$}
\STATE carry out some processing
\ENDFOR
\end{algorithmic}
```

produces

```
for i = 0 to 10 do
    carry out some processing
end for
```

and

```
\begin{algorithmic}[1]
\FORALL{$i$ such that $0 \leq i \leq 10$}
\STATE carry out some processing
\ENDFOR
\end{algorithmic}
```

produces

```

1: for all  $i$  such that  $0 \leq i \leq 10$  do
2:   carry out some processing
3: end for

```

### 3.3.1 The *to* Connective

As may be clear from the usage of loops above, we usually want to specify ranges over which a variable will assume values. To help make this typographically distinct, the `algorithmic` package now supports the **to** connective, which can be used like:

```

\begin{algorithmic}
\FOR{$i=0$ \TO $10$}
\STATE carry out some processing
\ENDFOR
\end{algorithmic}

```

to produce the output

```

for  $i = 0$  to 10 do
  carry out some processing
end for

```

## 3.4 The *while* Loop

The *while* loop takes the form

```
\WHILE{<condition>} <text> \ENDWHILE
```

For example,

```

\begin{algorithmic}
\WHILE{some condition holds}
\STATE carry out some processing
\ENDWHILE
\end{algorithmic}

```

produces

```

while some condition holds do
  carry out some processing
end while

```

## 3.5 The *repeat-until* Loop

The *repeat-until* loop takes the form.

```
\REPEAT <text> \UNTIL{<condition>}
```

For example,

```
\begin{algorithmic}
\REPEAT
\STATE carry out some processing
\UNTIL{some condition is met}
\end{algorithmic}
```

produces

```
repeat
    carry out some processing
until some condition is met
```

### 3.6 The Infinite Loop

The infinite loop takes the form.

```
\LOOP <text> \ENDLOOP
```

For example,

```
\begin{algorithmic}
\LOOP
\STATE this processing will be repeated forever
\ENDLOOP
\end{algorithmic}
```

produces

```
loop
    this processing will be repeated forever
end loop
```

### 3.7 The Logical Connectives

The connectives **and**, **or**, **xor** and **not** can be used in boolean expressions in the familiar, expected way:

```
<expression> \AND <expression>
<expression> \OR <expression>
<expression> \XOR <expression>
\nOT <expression>
```

according to their arity.<sup>1</sup> For example,

```
\begin{algorithmic}
```

---

<sup>1</sup>But there is nothing that prevents the user from violating the arity, from a syntactic point of view.

```
\IF{\NOT{($year \% 400 \XOR $year \% 100 \XOR $year \% 4)}}
\STATE{$year$ does not represent a leap year.}
\ENDIF
\end{algorithmic}
```

produces

```
if not (year mod 400 xor year mod 100 xor year mod 4) then
    year does not represent a leap year.
end if
```

### 3.8 The Precondition

The precondition (that must be met if an algorithm is to correctly execute) takes the form:

```
\REQUIRE <text>
```

For example,

```
\begin{algorithmic}
\REQUIRE{$x \neq 0$ and $n \geq 0$}
\end{algorithmic}
```

produces

**Require:**  $x \neq 0$  and  $n \geq 0$

### 3.9 The Postcondition

The postcondition (that must be met after an algorithm has correctly executed) takes the form:

```
\ENSURE <text>
```

For example,

```
\begin{algorithmic}
\ENSURE{$x \neq 0$ and $n \geq 0$}
\end{algorithmic}
```

produces

**Ensure:**  $x \neq 0$  and  $n \geq 0$

## 3.10 Returning Values

The `algorithmic` environment offers a special statement for explicitly returning values in algorithms. It has the syntax:

```
\RETURN <text>
```

For example,

```
\begin{algorithmic}
\RETURN $(x+y)/2$
\end{algorithmic}
```

produces

```
return (x + y)/2
```

### 3.10.1 The “true” and “false” Values

Since many algorithms have the necessity of returning *true* or *false* values, `algorithms`, starting with version 2006-06-02, includes the keywords `\TRUE` and `\FALSE`, which are intended to print the values in a standard fashion, like the following snippet of an algorithm to decide if an integer *n* is even or odd:

```
\begin{algorithmic}
\IF{$n$ is odd}
    \RETURN \TRUE
\ELSE
    \RETURN \FALSE
\ENDIF
\end{algorithmic}
```

The code above produces the following output:

```
if n is odd then
    return true
else
    return false
end if
```

## 3.11 Printing Messages

Another feature of the `algorithmic` environment is that it currently provides a standard way of printing values (which is an operation used enough to merit its own keyword). It has the syntax:

```
\PRINT <text>
```

For example,

```
\begin{algorithmic}
\PRINT \texttt{“Hello, World!”}
\end{algorithmic}
```

produces

```
print “Hello, World!”
```

### 3.12 Comments

Comments may be inserted at most points in an algorithm using the form:

```
\COMMENT{<text>}
```

For example,

```
\begin{algorithmic}
\STATE do something \COMMENT{this is a comment}
\end{algorithmic}
```

produces

```
do something {this is a comment}
```

Because the mechanisms used to build the various algorithmic structures make it difficult to use the above mechanism for placing comments at the end of the first line of a construct, the commands `\IF`, `\ELSIF`, `\ELSE`, `\WHILE`, `\FOR`, `\FORALL`, `\REPEAT` and `\LOOP` all take an optional argument which will be treated as a comment to be placed at the end of the line on which they appear. For example,

```
repeat {this is comment number one}
  if condition one is met then {this is comment number two}
    do something
  else if condition two is met then {this is comment number three}
    do something else
  else {this is comment number four}
    do nothing
  end if
until hell freezes over
```

### 3.13 An Example

The following example demonstrates the use of the `algorithmic` environment to describe a complete algorithm. The following input

```
\begin{algorithmic}
\REQUIRE $n \geq 0$
```

```

\ENSURE $y = x^n$
\STATE $y \leftarrow 1$
\STATE $X \leftarrow x$
\STATE $N \leftarrow n$
\WHILE{$N \neq 0$}
\IF{$N$ is even}
\STATE $X \leftarrow X \times X$
\STATE $N \leftarrow N / 2$
\ELSE[$N$ is odd]
\STATE $y \leftarrow y \times X$
\STATE $N \leftarrow N - 1$
\ENDIF
\ENDWHILE
\end{algorithmic}

```

will produce

**Require:**  $n \geq 0$   
**Ensure:**  $y = x^n$

```

 $y \leftarrow 1$ 
 $X \leftarrow x$ 
 $N \leftarrow n$ 
while  $N \neq 0$  do
    if  $N$  is even then
         $X \leftarrow X \times X$ 
         $N \leftarrow N/2$ 
    else { $N$  is odd}
         $y \leftarrow y \times X$ 
         $N \leftarrow N - 1$ 
    end if
end while

```

which is an algorithm for finding the value of a number taken to a non-negative power.

### 3.14 Options and Customization

There is a single option, `noend` that may be invoked when the `algorithmic` package is loaded. With this option invoked the `end` statements are omitted in the output. This allows space to be saved in the output document when this is an issue.

#### 3.14.1 Changing Indentation

In the spirit of saving vertical space (which is especially important when submitting a paper for a journal, where space is frequently limited for authors), the `algorithmic` environment offers, beginning with the version released in 2005-

05-08, a way to control the amount of indentation that is used by a given algorithm.

The amount of indentation to be used is given by the command

```
\algsetup{indent=length}
```

where *length* is any valid length used by TeX. The default value of the indentation used by the `algorithmic` environment is 1 em (for “backward compatibility reasons”), but a value of 2 em or more is recommended, depending on the publication. For example, the snippet

```
\algsetup{indent=2em}
\begin{algorithmic}[1]
    \STATE $a \leftarrow 1$
    \IF{$a$ is even}
        \PRINT ``$a$ is even''
    \ELSE
        \PRINT ``$a$ is odd''
    \ENDIF
\end{algorithmic}
```

produces

```
1: a  $\leftarrow$  1
2: if a is even then
3:     print ``a is even''
4: else
5:     print ``a is odd''
6: end if
```

while

```
\algsetup{indent=5em}
\begin{algorithmic}[1]
    \STATE $a \leftarrow 1$
    \IF{$a$ is even}
        \PRINT ``$a$ is even''
    \ELSE
        \PRINT ``$a$ is odd''
    \ENDIF
\end{algorithmic}
```

would produce

```
1: a  $\leftarrow$  1
2: if a is even then
3:     print ``a is even''
4: else
5:     print ``a is odd''
6: end if
```

The intended use of this option is to allow the author to omit the *end* (see Section 3.14 for details) statements without loosing readability, by increasing the amount of indentation to a suitable level.

### 3.14.2 Changing Line Numbering

As mentioned in Section 3 and illustrated in Section 3.14.1, `algorithms` already provides you with the possibility of numbering lines.

Starting with the version released in 2005-07-05, you can now change two aspects of line numbering: the size of the line numbers (which, by default, is `\footnotesize`) and the delimiter used to separate the line number from the code (which, by default, is `:`, i.e., a colon).

You can change the size of the line numbers using the command:

```
\algsetup{linenosize=size}
```

where `size` is any of the various commands provided by `LATEX` to change the size of the font to be used. Among others, useful values are `\tiny`, `\scriptsize`, `\footnotesize` and `\small`. Please see the complete list of sizes in your `LATEX` documentation.

As another frequently requested feature, you can change the delimiter used with the line numbers by issuing the command:

```
\algsetup{linenodelimiter=delimiter}
```

where `delimiter` is any “well-formed” string, including the empty string. With this command, you can change the colon to a period (.) by issuing the command

```
\algsetup{linenodelimiter=.}
```

or even omit the delimiter, by specifying the empty string or a space (`\` ), whatever seems best for your document.

As an example of such commands, the code produced by

```
\algsetup{
    linenosize=\small,
    linenodelimiter=.
}
\begin{algorithmic}[1]
    \STATE $i \leftarrow 10$
    \RETURN $i$
\end{algorithmic}
```

would be something like

1. `i`  $\leftarrow$  10
2. `return` `i`

### 3.14.3 Customization

In order to facilitate the use of this package with foreign languages, all of the words in the output are produced via redefinable macro commands. The default definitions of these macros are:

```
\newcommand{\algorithmicrequire}{\textbf{Require:}}
\newcommand{\algorithmicensure}{\textbf{Ensure:}}
\newcommand{\algorithmicend}{\textbf{end}}
\newcommand{\algorithmicif}{\textbf{if}}
\newcommand{\algorithmicthen}{\textbf{then}}
\newcommand{\algorithmicelse}{\textbf{else}}
\newcommand{\algorithmicelsif}{\textbf{else}\ \textbf{if}}
\newcommand{\algorithmicendif}{\textbf{end}\ \textbf{if}}
\newcommand{\algorithmicfor}{\textbf{for}}
\newcommand{\algorithmicforall}{\textbf{for all}}
\newcommand{\algorithmicdo}{\textbf{do}}
\newcommand{\algorithmicendfor}{\textbf{end}\ \textbf{for}}
\newcommand{\algorithmicwhile}{\textbf{while}}
\newcommand{\algorithmicendwhile}{\textbf{end}\ \textbf{while}}
\newcommand{\algorithmicloop}{\textbf{loop}}
\newcommand{\algorithmicendloop}{\textbf{end}\ \textbf{loop}}
\newcommand{\algorithmicrepeat}{\textbf{repeat}}
\newcommand{\algorithmicuntil}{\textbf{until}}
\newcommand{\algorithmicprint}{\textbf{print}}
\newcommand{\algorithmicreturn}{\textbf{return}}
\newcommand{\algorithmictrue}{\textbf{true}}
\newcommand{\algorithmicfalse}{\textbf{false}}
```

If you would like to change the definition of these commands to another content, then you should use, in your own document, the standard L<sup>A</sup>T<sub>E</sub>X command `renewcommand`, with an usage like this:

```
\renewcommand{\algorithmicrequire}{\textbf{Input:}}
\renewcommand{\algorithmicensure}{\textbf{Output:}}
```

**About the Way Comments Are Formatted** The formatting of comments is implemented via a single argument command macro which may also be redefined. The default definition is

```
\newcommand{\algorithmiccomment}[1]{\{#1\}}
```

and another option that may be interesting for users familiar with C-like languages is to redefine the comments to be

```
\renewcommand{\algorithmiccomment}[1]{// #1}
```

Comments produced this way would be like this:

$i \leftarrow i + 1$  // Increments  $i$

This second way to present comments may become the default in a future version of this package.

## 4 The algorithm Environment

### 4.1 General

When placed within the text without being encapsulated in a floating environment algorithmic environments may be split over a page boundary, greatly detracting from their appearance.<sup>2</sup> In addition, it is useful to have algorithms numbered for reference and for lists of algorithms to be appended to the list of contents. The `algorithm` environment is meant to address these concerns by providing a floating environment for algorithms.

### 4.2 An Example

To illustrate the use of the `algorithm` environment, the following text

```
\begin{algorithm}
\caption{Calculate  $y = x^n$ }
\label{alg1}
\begin{algorithmic}
\REQUIRE  $n \geq 0 \vee x \neq 0$ 
\ENSURE  $y = x^n$ 
\STATE  $y \leftarrow 1$ 
\IF{$n < 0$}
\STATE  $X \leftarrow 1 / x$ 
\STATE  $N \leftarrow -n$ 
\ELSE
\STATE  $X \leftarrow x$ 
\STATE  $N \leftarrow n$ 
\ENDIF
\WHILE{$N \neq 0$}
\IF{$N$ is even}
\STATE  $X \leftarrow X \times X$ 
\STATE  $N \leftarrow N / 2$ 
\ELSE[$N$ is odd]
\STATE  $y \leftarrow y \times X$ 
\STATE  $N \leftarrow N - 1$ 
\ENDIF
\ENDWHILE
\end{algorithmic}
\end{algorithm}
```

---

<sup>2</sup>This is the expected behaviour for floats in L<sup>A</sup>T<sub>E</sub>X. If you don't care about having your algorithm split between pages, then one option that you have is to ignore the `algorithm` environment.

produces Algorithm 1 which is a slightly modified version of the earlier algorithm for determining the value of a number taken to an integer power. In this case, provided the power may be negative provided the number is not zero.

---

**Algorithm 1** Calculate  $y = x^n$

---

**Require:**  $n \geq 0 \vee x \neq 0$

**Ensure:**  $y = x^n$

```

 $y \leftarrow 1$ 
if  $n < 0$  then
     $X \leftarrow 1/x$ 
     $N \leftarrow -n$ 
else
     $X \leftarrow x$ 
     $N \leftarrow n$ 
end if
while  $N \neq 0$  do
    if  $N$  is even then
         $X \leftarrow X \times X$ 
         $N \leftarrow N/2$ 
    else //  $N$  is odd
         $y \leftarrow y \times X$ 
         $N \leftarrow N - 1$ 
    end if
end while

```

---

The command `\listofalgorithms` may be used to produce a list of algorithms as part of the table contents as shown at the beginning of this document. An auxiliary file with a suffix of `.loa` is produced when this feature is used.

### 4.3 Options

The appearance of the typeset algorithm may be changed by use of the options: `plain`, `boxed` or `ruled` during the loading of the `algorithm` package. The default option is `ruled`.

The numbering of algorithms can be influenced by providing the name of the document component within which numbering should be recommended. The legal values for this option are: `part`, `chapter`, `section`, `subsection`, `subsubsection` or `nothing`. The default value is `nothing` which causes algorithms to be numbered sequentially throughout the document.

### 4.4 Customization

In order to facilitate the use of this package with foreign languages, methods have been provided to facilitate the necessary modifications.

The title used in the caption within `algorithm` environment can be set by use of the standard `\floatname` command which is provided as part of the `float` package which was used to implement this package. For example,

```
\floatname{algorithm}{Procedure}
```

would cause **Procedure** to be used instead of **Algorithm** within the caption of algorithms.

In a manner analogous to that available for the built in floating environments, the heading used for the list of algorithms may be changed by redefining the command `listalgorithmname`. The default definition for this command is

```
\newcommand{\listalgorithmname}{List of Algorithms}
```

#### 4.4.1 Placement of Algorithms

One important fact that many users may not have noticed is that the `algorithm` environment is actually built with the `float` package and `float`, in turn, uses David Carlisle's `here` style option. This means that the floats generated by the `algorithm` environment accept a special option, namely, `[H]`, with a capital 'H', instead of the usual 'h' offered by plain  $\text{\LaTeX}$ .

This option works as a stronger request of "please put the float here": instead of just a suggestion for  $\text{\LaTeX}$ , it actually means "put this float HERE", which is something desired by many. The two algorithms typeset in this document use this option.

**Warning** You can't use the 'H' positioning option together with the usual 'h' (for "here"), 'b' (for "bottom") etc. This is a limitation (as far as I know) of the `float.sty` package.

## 5 Labels and References in Algorithms

With the release of 2005-07-05, now `algorithmic` accepts labels and references to specific lines of a given algorithm, so you don't have to hardcode the line numbers yourself when trying to explain what the code does in your texts. Thanks to Arnaud Legrand for the suggestion and patch for this highly missed feature.

An example of its use is shown in Algorithm 2.

---

**Algorithm 2** Calculate  $y = x^n$ 

---

**Require:**  $n \geq 0 \vee x \neq 0$   
**Ensure:**  $y = x^n$

```
1:  $y \leftarrow 1$ 
2: if  $n < 0$  then
3:    $X \leftarrow 1/x$ 
4:    $N \leftarrow -n$ 
5: else
6:    $X \leftarrow x$ 
7:    $N \leftarrow n$ 
8: end if
9: while  $N \neq 0$  do
10:  if  $N$  is even then
11:     $X \leftarrow X \times X$ 
12:     $N \leftarrow N/2$ 
13:  else
14:     $y \leftarrow y \times X$ 
15:     $N \leftarrow N - 1$ 
16:  end if
17: end while
```

---

See that, in line 10, we deal with the case of  $N$  being even, while, in line 13, we give treatment to the case of  $N$  being odd. The numbers you see on this document were generated automatically from the source document.

## 6 Issues Between algorithms and tocbibind or memoir

It has been discussed in late 2005 that `algorithms` may have bad interactions with the `tocbibind` or the `memoir` package (which includes `tocbibind`).

A workaround has been suggested for the problem. After including something like

```
\usepackage[nottoc]{tocbibind}
```

in the preamble of your document, you can put, after `\begin{document}`, the following snippet of code:

```
\renewcommand{\listofalgorithms}{\begingroup
  \tocfile{List of Algorithms}{loa}
\endgroup

\makeatletter
\let\l@algorithm\l@figure
\makeatother
```

which should make the command `\listofalgorithms` work as expected.

## 7 Hints for Typesetting Algorithms

Here are some short hints on typesetting algorithms:

- Don't overcomment your pseudo-code. If you feel that you need to comment too much, then you are probably doing something wrong: you should probably detail the inner workings of the algorithm in regular text rather than in the pseudo-code;
- Similarly, don't regard pseudo-code as a low-level programming language: *don't pollute your algorithms* with punctuation marks like semi-colons, which are necessary in C, C++ and Java, but not in pseudo-code. Remember: your readers *are not* compilers;
- Always document what the algorithm receives as an input and what it returns as a solution. Don't care to say in the \REQUIRE or in the \ENSURE commands *how* the algorithm does what it does. Put this in the regular text of your book/paper/lecture notes;
- If you feel that your pseudo-code is getting too big, just break it into sub-algorithms, perhaps abstracting some tasks. Your readers will probably thank you.

Of course, you should follow those hints with common sense. Well, anything should be done with common sense.